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μ-πrα Closed Sets in Bigeneralized Topological Spaces

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Abstract
The aim of the paper is to introduce the concept of $\mu_{(m,n)}$-πrα closed sets in bigeneralized topological spaces and study some of their properties. We also introduce the notion of $\mu_{(m,n)}$-πrα continuous function and $\mu_{(m,n)}$-πrα $T_{1/2}$ spaces on bigeneralized topological spaces and investigate some of their properties.
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I. 1.Introduction

Á.Császár [3] introduced the concepts of generalized neighborhood systems and generalized topological spaces. He also introduced the notions of continuous functions and associated interior and closure operators on generalized neighborhood systems and generalized topological spaces. Also in [4] he investigated the notions of μ-α-open sets, μ-semi open sets, μ- pre open sets and μ- β open sets in generalized topological space.

W.Dungthaisong, C.Boonpok and C.Viriyapong [9] introduced the concepts of bigeneralized topological spaces and studied (m, n)- closed sets and (m, n)- open sets in bigeneralized topological spaces. Also several authors were further extended the concept of various types of closed sets [2, 5, 6, 7] in bigeneralized topological spaces.

In this paper, we introduce the notion of $\mu_{(m,n)}$-πrα closed sets, $\mu_{(m,n)}$-πrα continuous functions and $\mu_{(m,n)}$-πrα $T_{1/2}$ spaces in bigeneralized topological spaces and investigate some of their properties.

II. Preliminaries

We recall some basic concepts and results.

Let X be a nonempty set and let exp(X) be the power set of X. μ⊂exp(X) is called a generalized topology [3](briefly, GT) on X, if ∅ ∈μ and unions of elements of μ belong to μ. The pair (X, μ) is called a generalized topological space (briefly, GTS). The elements of μ are called μ-open subsets of X and the complements are called μ-closed sets. If (X, μ) is a GTS and A⊆X, then the interior of (denoted by $i_\mu(A)$) is the union of all G⊆A, G∈μ and the closure of A (denoted by $c_\mu(A)$) is the intersection of all μ-closed sets containing A. Note that $c_\mu(A) = X-i_\mu(X-S)$ and $i_\mu(A) = X - c_\mu(X-A)$ [3].

Definition 2.1[4] Let (X, μ_x) be a generalized topological space and A⊆ X. Then A is said to be

(i) μ- semi open if $A\subseteq c_\mu(i_\mu(A))$.
(ii) μ- pre open if $A\subseteq i_\mu(c_\mu(A))$.
(iii) μ-α-open if $A\subseteq i_\mu(c_\mu(i_\mu(A)))$.
(iv) μ-β-open if $A\subseteq c_\mu(i_\mu(c_\mu(A)))$.
(v) μ-r-open [8] if $A = i_\mu(c_\mu(A))$
(vi) μ-rα-open [1] if there is a μ-r-open set U such that $U\subset A\subset c_\alpha(U)$.

Definition 2.2 [1] Let (X, μ_x) be a generalized topological space and A⊆ X. Then A is said to be μ-πrα closed set if $c_\pi(A) \subseteq U$ whenever A⊆U and U is μ-rα-open set. The complement of μ-πrα closed set is said to be μ-πrα open set.

The complement of μ-semi open (μ-pre open, μ-α-open, μ-β-open, μ-r-open, μ-rα-open) set is called μ- semi closed (μ- pre closed, μ-α- closed, μ-β-closed, μ-r- closed, μ-rα-closed) set.

Let us denote the class of all μ-semi open sets, μ-pre open sets, μ-α-open sets, μ-β-open sets, and μ-πrα open sets on X by $\sigma(\mu_x)$ (σ for short), $\pi(\mu_x)$ (π for short), $\alpha(\mu_x)$ (α for short), $\beta(\mu_x)$ (β for short) and $\pi p(\mu_x)$ (πp for short)respectively. Let μ be a generalized topology on a non empty set X and S⊆X. The μ-α-closure (resp. μ-semi closure, μ-pre closure, μ-β-closure, μ-πrα-closure) of a subset S of X denoted by $c_\alpha(S)$ (resp. $c_\sigma(S)$, $c_\pi(S)$, $c_\beta(S)$, $c_{\pi p}(S)$) is the intersection of μ-α-closed(resp. μ- semi closed, μ- pre closed, μ-β-closed, μ-πrα closed) sets including S. The μ-α-interior (resp. μ-semi interior, μ-pre interior, μ-β-interior, μ-πrα-interior) of a subset S of X denoted by $i_\alpha(S)$ (resp. $i_\sigma(S)$, $i_\pi(S)$, $i_\beta(S)$, $i_{\pi p}(S)$) is the union of μ-α-open (resp. μ- semi open, μ- pre open, μ-β-open, μ-πrα open) sets contained in S.

Definition 2.3 [2] Let X be a nonempty set and μ_1, μ_2 be generalized topologies on X. A triple (X, μ_1, μ_2) is said to be a bigeneralized topological space.

Remark 2.4 Let (X, μ_1, μ_2) be a bigeneralized topological space and A be a subset of X. The closure of A and the interior of A with respect to μ_m are denoted by cμ_m(A) and iμ_m(A) respectively for m = 1,2. The family of all μ_m- closed set is denoted by the symbol F_n.

Definition 2.5 [2**]** A subset A of bigeneralized topological space (X, μ_1,μ_2) is called (m, n)- closed if cμ_m(cμ_n(A))=A, where m, n = 1, 2 and m≠n. The complements of (m, n) closed sets is called (m, n)-open.

Proposition 2.6 [2**]** Let (X, μ_1, μ_2) be a bigeneralized topological space and A be a subset of X. Then A is (m, n)- closed if and only if A is both μ- closed in (X, μ_m) and (X, μ_n).

Definition 2.7 [9] A subset A of a bigeneralized topological space (X,μ_1, μ_2) is said to be (m, n)-generalized closed (briefly $\mu_{(m,\ n)}$- closed) set if cμ_m(A) ⊆ U whenever A⊆ U and U is μ_m- open set in X, where m, n= 1,2 and m≠n. The complement of $\mu_{(m,n)}$- closed set is said to be (m, n)- generalized open (briefly $\mu_{(m,n)}$-open) set.

Definition 2.8 [9] A bigeneralized topological space (X, μ_1, μ_2) is said to be $\mu_{(m,n)}$-$T_{1/2}$-space if every $\mu_{(m,n)}$- closed set is μ_n-closed, where m, n= 1,2 and m≠n.

Definition 2.9[9] Let (X,μ_x^1, μ_x^2) and (Y, μ_y^1, μ_y^2) be bigeneralized topological spaces. A function f:(X, μ_x^1, μ_x^2) → (Y, μ_y^1, μ_y^2)is said to be (m, n)-generalized continuous(briefly, $g_{(m,n)}$- continuous) if f^{-1}(F) is $\mu_{(m,n)}$- closed in X for every μ_n- closed set F of Y, where m, n = 1,2 and m≠n.
A function f: (X,μ_x^1, μ_x^2) →(Y,μ_y^1, μ_y^2)is said to be pairwise continuous if f:(X,μ_x^1)→(Y, μ_y^1) and f:(X, μ_x^2)→(Y, μ_y^2) are continuous.

Definition 2.10 [9] Let (X, μ_x^1,μ_x^2) and (Y, μ_y^1, μ_y^2) be bigeneralized topological spaces. A function f: (X,μ_x^1, μ_x^2) →(Y,μ_y^1, μ_y^2) is said to be $g_{(m,n)}$- irresolute if f^{-1}(F) is $\mu_{(m,n)}$- closed in X for every $\mu_{(m,n)}$- closed set F in Y, where m, n = 1,2 and m≠n.

III. $\mu_{(m, n)}$-□ r□ closed sets

In this section, we introduce $\mu_{(m,\ n)}$- πrα closed sets in bigeneralized topological spaces and study some of their properties.

Definition 3.1 A subset A of a bigeneralized topological space (X, μ_1,μ_2) is said to be $\mu_{(m,\ n)}$- πrα closed if cπ_n (A)⊆U whenever A⊆ U and U is μ_m-rα open set in X, where m, n = 1,2 and m≠n. The complement of a $\mu_{(m,\ n)}$- πrα closed set is said to be $\mu_{(m,\ n)}$-πrα open set.

We denote the family of all $\mu_{(m,\ n)}$- πrα closed (resp. $\mu_{(m,\ n)}$- πrα open) set in (X, μ_1,μ_2) is $\mu_{(m,\ n)}$- πrαC(X) (resp. $\mu_{(m,\ n)}$- πrαO(X)), where m, n= 1,2 and m ≠n.

Theorem 3.2 (i) Every (m, n)- closed set is $\mu_{(m,\ n)}$-πrα closed.
(ii) Every μ_n- closed set is $\mu_{(m,\ n)}$-πrα closed.
Proof: (i) Let A⊆ U and U is μ_m-rα open set. Since A is (m, n)- closed set then cμ_m(cμ_n (A)) =A. Therefore cπ_m(cπ_n(A)) ⊆ cμ_m(cμ_n(A)) = A ⊆U.Since cπ_n(A) ⊆cπ_m(cπ_n(A)⊆ U, we get cπ_n(A)⊆ U. Hence A is $\mu_{(m,\ n)}$-πrα closed.
(ii) Let A be a μ_n-closed set and U be a μ_m-rα open set containing A. Then cμ_n(A) = A⊆U. Since cπ_n(A) ⊆cμ_n(A), we get cπ_n(A)⊆U. Therefore we get A is $\mu_{(m,\ n)}$-πrα closed.

The converse of the above theorem is not true as seen from the following example.

Example 3.3 Let X= {a, b, c}.Consider the two topologies μ_1= {∅, {a}, {a, b}, X} and μ_2= **{**∅, {c}, {b, c}}. Then the subset{c} is $\mu_{(1,2)}$-πrα closed but not (1,2)- closed and μ_2-closed.

Remark 3.4 The concepts $\mu_{(m,\ n)}$- closed and $\mu_{(m,\ n)}$-πrα closed are independent notions. This can be seen from the following examples.

Example 3.5 $\mu_{(1,\ 2)}$-πrα closed ⇏$\mu_{(1,2)}$-closed.
Let X = {a, b, c}. Consider the two topologies μ_1= {∅, {a}, {a, b}, X} and μ_2= **{**∅, {c}, {b, c}} on X. Then ∅ is $\mu_{(1,\ 2)}$- πrα closed but not $\mu_{(1,\ 2)}$- closed.

Example 3.6 $\mu_{(1,\ 2)}$- closed ⇏$\mu_{(1,2)}$-πrα closed.
Let X = {a, b, c, d}. Consider the two topologies μ_1= {∅, {a},{c}, {a, c}, {a, b, c}, X} and μ_2= **{**∅, {c}, {b, c}} on X. Then {b, c} and {b, d} are is $\mu_{(1,\ 2)}$-closed but not $\mu_{(1,\ 2)}$-πrα closed.

Remark 3.7 The intersection of two $\mu_{(m,\ n)}$-πrα closed sets need not be a $\mu_{(m,\ n)}$-πrα closed set as seen from the following example.

Example 3.8 Let X = {a, b, c, d}. Consider the two topologies μ_1= {∅, {a},{c}, {a, c}, {a, b, c}, X} and μ_2= **{**∅, {c}, {b, c}} on X. Then {a, c}and {c, d}are is $\mu_{(1,\ 2)}$-πrα closed but {a, c}∩{c, d} = {c} is not $\mu_{(1,\ 2)}$-πrα closed.

Remark 3.9 The union of two $\mu_{(m,\ n)}$-πrα closed sets need not be a $\mu_{(m,\ n)}$-πrα closed set as seen from the following example.

Example 3.10 Let X = {a, b, c}. Consider the two topologies μ_1= {∅, {b}, {a, c}, {b, c}, X} and μ_2= **{**∅, {b, c}, {a, c}, X} on X. Then {a}and {c}are is $\mu_{(1,\ 2)}$-

πrα closed but $\{a\}\cup\{c\}=\{a, c\}$ is not $\mu_{(1, 2)}$-πrα closed.

Proposition 3.11 For each element x of a bigeneralized topological space (X, μ_1, μ_2), {x} is μ_m-rα closed or X – {x} is $\mu_{(m, n)}$-πrα closed, where m, n = 1, 2 and m ≠ n.
Proof: Let x ∈X and the singleton{x} be not μ_m-rα closed. Then X –{x} is not μ_m-rα open.
If X is μ_m-rα open then X is the only μ_m-rα open set which contain X− {x}and so X− {x}is $\mu_{(m, n)}$-πrα closed. And if X is not μ_m-rα open then X− {x} is $\mu_{(m, n)}$-πrα closed as there is no μ_m-rα open set which contains X− {x}and hence the condition is satisfied vacuously. .

Proposition 3.12 Let (X, μ_1, μ_2) be a bigeneralized topological space. Let A⊆X be a $\mu_{(m, n)}$-πrα closed subset of X, then $c\pi_n$(A)−A does not contain any non empty μ_m-rα closed set, where m, n= 1,2 and m≠n.
Proof: Let A be a $\mu_{(m, n)}$- πrα closed set and F ≠ ∅ is μ_m-rα closed such that F⊆$c\pi_n$(A)−A.Then F⊆X−A implies A⊆X−F. Since A is $\mu_{(m, n)}$-πrα closed, $c\pi_n$(A)⊆X−F and hence F⊆X−$c\pi_n$(A).
That is F⊆$c\pi_n$(A)∩(X−$c\pi_n$(A))=∅.
Therefore $c\pi_n$(A)−A does not contain any nonempty μ_m-rα closed set.

Proposition 3.13 Let μ_1 and μ_2 be generalized topologies on X. If A is $\mu_{(m, n)}$-πrα closed set, then $c\pi_n$({x})∩A≠ ∅ holds for each x∈$c\pi_n$(A), where m, n = 1, 2and m≠n.
Proof: Let x∈$c\pi_n$(A). Suppose that $c\pi_m$({x})∩A= ∅. Then A⊆X−$c\pi_m$({x}).Since A is $\mu_{(m, n)}$-πrα closed and X−$c\pi_m$({x}) is μ_m-rα open. We get $c\pi_n$(A)⊆X−$c\pi_m$({x}). Hence $c\pi_n$(A)∩$c\pi_m$({x})=∅, which is a contradiction to our assumption. Therefore $c\pi_m$({x})∩A ≠ ∅.

Proposition 3.14 If A is $\mu_{(m, n)}$-πrα closed set of (X, μ_1,μ_2) such that A⊆B⊆ $c\pi_n$(A) then B is $\mu_{(m, n)}$ −πrα closed set, where m, n= 1,2 and m≠n.
Proof: Let A be $\mu_{(m, n)}$-πrα closed set and A⊆B⊆$c\pi_n$(A). Let B⊆ U and U is μ_m-rα open. Then A⊆U. Since A is $\mu_{(m, n)}$-πrα closed, we have $c\pi_n$(A)⊆U.Since B⊆$c\pi_n$(A), then $c\pi_n$(B)⊆$c\pi_n$(A)⊆U. Hence B is $\mu_{(m, n)}$-πrα closed.

Remark 3.15 $\mu_{(1, 2)}$-πrαC(X) is generally not equal to $\mu_{(2, 1)}$-πrαC(X), as can be seen from the following example.
Let X = {a, b, c}. Consider the two topologies μ_1= {∅, {a}, {a, b}, X} and μ_2= {∅, {b}, {a, c},{b, c}, X} on X. Then $\mu_{(1, 2)}$-πrαC(X)= {∅,X,{a},{b},{c},{a, b},{b, c},{a, c}}and $\mu_{(2, 1)}$-πrαC(X) = {∅, X,{b},{c},{a, b},{b, c}}.Thus $\mu_{(1, 2)}$-πrαC(X)≠$\mu_{(2, 1)}$-πrαC(X).

Proposition 3.16 Let μ_1 and μ_2 be generalized topologies on X if μ_1⊆μ_2, then $\mu_{(2, 1)}$-πrαC(X)⊆$\mu_{(1, 2)}$-πrαC(X).
Proof: Let A be a $\mu_{(2, 1)}$-πrα closed set and U be a μ_1-rα open set containing A. Since μ_1⊆μ_2, we have $c\pi_2$(A)⊆$c\pi_1$(A) and μ_1-C(X)⊆μ_2-C(X). Since A ∈$\mu_{(2, 1)}$-πrαC(X), $c\pi_1$(A)⊆U. Therefore $c\pi_2$(A)⊆U, U is μ_1-rα open. Thus A∈$\mu_{(1, 2)}$-πrαC(X).

Proposition 3.17 If A be a subset of a bigeneralized topological space (X, μ_1,μ_2) is $\mu_{(m, n)}$-πrα closed then $c\pi_n$(A)−A is $\mu_{(m, n)}$-πrα open, where m, n=1, 2 and m≠n.
Proof: Suppose that A is $\mu_{(m, n)}$-πrα closed. Let X−($c\pi_n$(A)−A)⊆U and U is μ_m-rα open. Then (X−U)⊆($c\pi_n$(A)−A) and X−U is μ_m-rα closed. Thus by Proposition 3.12 $c\pi_n$(A)−A does not contain non empty μ_m-rα closed.
Consequently X− U = ∅, then X= U. Therefore $c\pi_n$(X−($c\pi_n$(A)−A))⊆U.
So we obtain X−($c\pi_n$(A)−A) is $\mu_{(m, n)}$-πrα closed. Hence $c\pi_n$(A)−A is $\mu_{(m, n)}$-πrα open.

IV. $\mu_{(m, n)}$-π rα continuous functions

In this section, we introduce $\mu_{(m, n)}$- πrα continuous functions and $\mu_{(m, n)}$-πrα $T_{1/2}$ in bigeneralized topological spaces and study their properties.

Definition 4.1 Let (X,μ_x^1, μ_x^2) and (Y,μ_y^1, μ_y^2) be bigeneralized topological spaces. A function f: (X,μ_x^1,μ_x^2) →(Y,μ_y^1, μ_y^2) is said to be $\mu_{(m, n)}$-πrα continuous if f^{-1}(F) is $\mu_{(m, n)}$-πrα closed in X for every μ_n-closed set F of Y where m, n= 1,2 and m≠n.

Definition 4.2 Let (X,μ_x^1,μ_x^2) and (Y,μ_y^1, μ_y^2) be bigeneralized topological spaces.
A function f: (X,μ_x^1, μ_x^2) →(Y,μ_y^1, μ_y^2) is said to be $\mu_{(m, n)}$-πrα irresolute if f^{-1}(F) is $\mu_{(m, n)}$-πrα closed in X for every $\mu_{(m, n)}$-πrα closed set F in Y where m, n= 1,2 and m≠n.

Theorem 4.3 Every pairwise continuous function is $\mu_{(m,n)}$-πrα continuous.
Proof: Let f: ((X,μ_x^1, μ_x^2) →(Y,μ_y^1, μ_y^2) be pairwise continuous. Let F be a μ_n-closed set in Y.
Then f^{-1}(F) is μ_n-closed in X. Since every μ_n-closed is $\mu_{(m, n)}$-πrα closed, where m, n = 1, 2 and m ≠n.We have f is $\mu_{(m, n)}$-πrα continuous.

Theorem 4.4 For an injective function f: (X,μ_x^1, μ_x^2) →(Y,μ_y^1, μ_y^2) the following properties are equivalent.
(i) f is $\mu_{(m, n)}$-πrα continuous.
(ii) For each x∈X and for every μ_n- open set V containing f(x), there exists a $\mu_{(m, n)}$-πrα open set U containing x such that f(U) ⊆V.

(iii) $f(c\mu_x^n\ (A)) \subseteq c\mu_x^n$ (f(A)) for every subset A of X.

(iv) $c\mu_x^n\ (f-1(B)) \subseteq f-1(c\mu_y^n$ (B)) for every subset B of Y.

Proof: (i) ⇒(ii) Let x∈X and V be a μ_n-open subset of Y containing f(x). Then by (i), $f^{-1}(V)$ is $\mu_{(m,\ n)}$-πrα open of X containing x. If U= $f^{-1}(V)$ then f(U) ⊆V.

(ii)⇒(iii) Let A be a subset of X and f(x) $\notin c\mu_y^n$ (f(A)). Then, there exists a μ_n-open subset V of Y containing f(x) such that V∩f(A)= ∅. Then by (ii), there exist a $\mu_{(m,\ n)}$-πrα open set U such that f(x)∈f(U)⊆V. Hence f(U)∩f(A) = $\emptyset$ *implies* U∩A = $\emptyset. Consequently, x \notin c\mu_x^n(A)\ and\ f(x) \notin c\mu_x^n(A)$.

(iii)⇒(iv) Let B be a subset of Y.

By (iii) we have $f(c\mu_x^n(f-1(B))) \subseteq c\mu_y^n(f(f-1(B)))$.

$$Thus\ (c\mu_x^n(f-1(B))) \subseteq\ f-1(c\mu_y^n(B)).$$

$(iv) \Rightarrow (i) Let\ A\ be\ a\ \mu n - closed\ subset\ of\ Y, Let\ U\ be\ a\ \mu m - r\alpha\ open\ subset\ of\ X\ such\ that\ f-1(A) \subseteq U. Since\ c\mu_y^n(A) = \ A\ and\ by\ (iv)\ c\pi_x^n(f-1(A)) \subseteq c\mu_x^n(f-1(A)) \subseteq f-1(c\mu_y^n(A)) \subseteq f-1(A) \subseteq$

Hence $f^{-1}(A)$ is $\mu_{(m,\ n)}$-πrα closed for each μ_n-closed set A in Y. Therefore f is $\mu_{(m,\ n)}$-πrα continuous.

Definition 4.5 A bigeneralized topological space (X,μ_x^1, μ_x^2) is said to be $\mu_{(m,\ n)}$-πrα $T_{1/2}$ space if for every $\mu_{(m,\ n)}$-πrα closed set is μ_n-closed where m, n = 1, 2 and m ≠n.

Proposition 4.6 Let f:(X,μ_x^1, μ_x^2) →(Y,μ_y^1, μ_y^2) and g:(Y,μ_y^1, μ_y^2) →(Z,μ_z^1, μ_z^2) be functions, the following properties hold.

(i) If f is $\mu_{(m,\ n)}$-πrα irresolute and $\mu_{(m,\ n)}$-πrα continuous then g∘f is $\mu_{(m,\ n)}$-πrα continuous.

(ii) If f and g are $\mu_{(m,\ n)}$-πrα irresolute then g∘f is $\mu_{(m,\ n)}$-πrα irresolute.

(iii) Let (Y,μ_y^1, μ_y^2) be a $\mu_{(m,\ n)}$-πrα $T_{1/2}$ space. If f and g are $\mu_{(m,\ n)}$-πrα continuous then g∘f is $\mu_{(m,\ n)}$-πrα continuous.

(iv) If f is $\mu_{(m,\ n)}$-πrα continuous and g is pairwise continuous then g∘f is $\mu_{(m,\ n)}$-πrα continuous.

Proof: (i) Let F be a μ_n-closed subset of Z. Since g is $\mu_{(m,\ n)}$-πrα continuous then $g^{-1}(F)$ is $\mu_{(m,\ n)}$-πrα closed subset of Y. Since f is $\mu_{(m,\ n)}$-πrα irresolute, then $(g∘f)^{-1}(F)= f^{-1}(g^{-1}(F))$ is $\mu_{(m,\ n)}$-πrα continuous.

(ii) Let F be a $\mu_{(m,\ n)}$- πrα closed subset of Z. Since g is $\mu_{(m,\ n)}$-πrα irresolute, then $g^{-1}(F)$ is $\mu_{(m,\ n)}$-πrα closed subset of Y. Since f is $\mu_{(m,\ n)}$-πrα irresolute, then $(g∘f)^{-1}(F)= f^{-1}(g^{-1}(F))$ is $\mu_{(m,\ n)}$-πrα closed subset of X. Hence g∘f is $\mu_{(m,\ n)}$-πrα irresolute.

(iii) Let F be a μ_n-closed subset of Z. Since g is $\mu_{(m,\ n)}$-πrα continuous, then $g^{-1}(F)$ is $\mu_{(m,\ n)}$-πrα closed subset of Y. Since (Y,μ_y^1, μ_y^2) is a $\mu_{(m,\ n)}$-πrα $T_{1/2}$ space then $g^{-1}(F)$ is μ_n-closed subset of Y. Since f is $\mu_{(m,\ n)}$-πrα continuous then $(g∘f)^{-1}(F)= f^{-1}(g^{-1}(F))$ is $\mu_{(m,\ n)}$- πrα closed subset of X. Hence g∘f is $\mu_{(m,\ n)}$-πrα continuous.

(iv) Let f be $\mu_{(m,\ n)}$-πrα continuous and g be a pairwise continuous. Let F be μ_n-closed in Z. Since g is pairwise continuous, $g^{-1}(F)$ is μ_n-closed. Since f is $\mu_{(m,\ n)}$-πrα continuous, $f^{-1}(g^{-1}(F))$ is $\mu_{(m,\ n)}$-πrα closed in X , m, n = 1, 2 and m≠n. Therefore g∘f is $\mu_{(m,\ n)}$-πrα continuous.

Theorem 4.7 A bigeneralized topological space is $\mu_{(m,\ n)}$-πrα $T_{1/2}$ iff {x}is μ_n-open or μ_m-rα closed for each x∈X, where m, n = 1,2 and m≠n.

Proof: Suppose that {x} is not μ_m-rα closed. Then X−{x} is $\mu_{(m,\ n)}$-πrα closed by Proposition 3.11. Since X is $\mu_{(m,\ n)}$-πrα $T_{1/2}$ space, X−{x}is μ_n-closed. Hence {x} is μ_n-open.

Conversely,

Let F be a $\mu_{(m,\ n)}$-πrα closed set. By assumption {x} is μ_n-open or μ_m-rα closed for any x∈ $c\pi_n$(F).

Case i: Suppose that {x} is μ_n-open. Since {x}∩F≠ ∅, we have x∈F.

Case ii: Suppose {x} is μ_m-rα closed. If x∉F, then {x}⊆ $c\pi_n$(F)−F.

This is a contradiction to the Proposition 3.12. Therefore x∈F. Thus in both cases, we conclude that F is μ_n-closed. Hence (X,μ_x^1, μ_x^2) is a $\mu_{(m,\ n)}$-πrα $T_{1/2}$ space.

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